

	P.R.Government College (Autonomous) KAKINADA	Program&Semester III B.Sc. (V Sem)			
CourseCode MAT-601A / 5231	TITLEOFTHECOURSE VI A- Numerical Methods				
Teaching	HoursAllocated:60(Theory)	L	T	P	C
Pre-requisites:	Advanced Calculus, Linear Algebra and Differential Equations	4	1	-	4

Course Objectives:

This course will cover the classical fundamental topics in numerical methods such as, approximation, numerical integration, numerical linear algebra, solution of nonlinear algebraic systems and solution of ordinary differential equations.

Course Outcomes:

On Completion of the course, the students will be able o-	
C01	Understand various finite difference concepts and interpolation methods.
C02	Work out numerical differentiation and integration whenever and wherever routine methods are not applicable.
C03	Find numerical solutions of ordinary differential equations by using various numerical methods.
C04	Analyze and evaluate the accuracy of numerical methods.

Course with focus on employability/entrepreneurship /Skill Development modules



Skill Development		Employability		Entrepreneurship	
-------------------	--	---------------	--	------------------	--

Unit – 1: Finite Differences and Interpolation with Equal intervals

1. Introduction, Forward differences, Backward differences, Central Differences, Symbolic relations, nth Differences of Some functions,
2. Advancing Difference formula, Differences of a Polynomial.
3. Newton's formulae for interpolation.

Unit – 2: Interpolation with Equal and Unequal intervals

1. Central Difference Interpolation Formulae.

Gauss's Forward interpolation formulae, Gauss's backward interpolation formulae, Stirling's formula, Bessel's formula.

2. Interpolation with unevenly spaced points, divided differences and properties, Newton's divided differences formula.

3. Lagrange's interpolation formula, Lagrange's Inverse interpolation formula.

Unit – 3: Numerical Differentiation

1. Derivatives using Newton's forward difference formula, Newton's back ward difference formula,

2. Derivatives using central difference formula, Stirling's interpolation formula,

3. Newton's divided difference formula, Maximum and minimum values of a tabulated function.

Unit – 4: Numerical Integration

1. General quadrature formula one errors, Trapezoidal rule,

2. Simpson's $1/3$ - rule, Simpson's $3/8$ – rule and Weddle's rules,

3. Euler – McLaurin Formula of summation and quadrature, The Euler transformation.

Unit – 5: Numerical solution of ordinary differential equations

1. Introduction, Solution by Taylor's Series,

2. Picard's method of successive approximations,

3. Euler's method, Modified Euler's method, Runge – Kutta methods.

III. References:

1. S.S.Sastry, Introductory Methods of Numerical Analysis, Prentice Hall of India Pvt. Ltd., New Delhi-110001, 2006.

2. P.Kandasamy, K.Thilagavathy, Calculus of Finite Differences and Numerical Analysis. S. Chand & Company, Pvt. Ltd., Ram Nagar, New Delhi-110055.

3. R.Gupta, Numerical Analysis, Laxmi Publications (P) Ltd., New Delhi.

4. H.C Saxena, Finite Differences and Numerical Analysis, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

5. S.Ranganatham, Dr.M.V.S.S.N.Prasad, Dr.V.Ramesh Babu, Numerical Analysis, S. Chand & Company Pvt. Ltd., Ram Nagar, New Delhi-110055.

6. Web resources suggested by the teacher and college librarian including reading material.

IV. Co-Curricular Activities: A) Mandatory:

1. For Teacher: Teacher shall train students in the following skills for 15 hours, by taking relevant outside data (Field/Web).

1. Applications of Newton's forward and back ward difference formulae.

2. Applications of Gauss forward and Gauss back ward, Stirling's and Bessel's formulae.
3. Applications of Newton's divided differences formula and Lagrange's interpolation formula.
4. Various methods to find the approximation of a definite integral.
5. Different methods to find solutions of Ordinary Differential Equations.

2. For Student: Fieldwork/Project work; Each student individually shall undertake Fieldwork/Project work and submit a report not exceeding 10 pages in the given format on the work done in the areas like the following, by choosing any one of the aspects.

1. Collecting the data from the identified sources like Census department or Electricity department, by applying the Newton's, Gauss and Lagrange's interpolation formula, making observations and drawing conclusions. (Or)
2. Selection of some region to find the area by applying Trapezoidal rule, Simpson's $1/3$ – rule, Simpson's $3/8$ – rule, and Weddle's rules. Comparing the solutions with analytical solution and concluding which one is the best method. (Or)
3. Findingsolutionof the ODE by Taylor's Series, Picard's method of successive approximations, Euler's method, Modified Euler's method, Runge–Kutta methods. Comparing the solutions with analytical solution, selecting the best method.

3. Max. Marks for Fieldwork/Project work Report: 05.

4. Suggested Format for Fieldwork/Project work Report: Title page, Student Details, Index page, Stepwise work-done, Findings, Conclusions and Acknowledgements.

5. Unit tests (IE).

b) Suggested Co-Curricular Activities:

1. Assignments/collection of data, Seminar, Quiz, Group discussions/Debates
2. Visits to research organizations, Statistical Cells, Universities, ISI etc.
3. Invited lectures and presentations on related topics by experts in the specified area .

CO-PO

Mapping:

(1:Slight[Low];

2:Moderate[Medium];

3:Substantial[High],

'-':NoCorrelation)

	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PS01	PS02	PS03
C01	3	3	2	3	3	3	1	2	2	3	2	3	2
C02	3	2	3	3	2	3	3	1	3	3	3	2	1
C03	2	3	2	3	2	3	2	2	2	3	2	2	3
C04	3	2	3	2	3	2	3	3	2	1	3	1	2

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-V : PAPER-VI A

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	Finite Differences and Interpolation with Equal intervals	2	1	20
II	Interpolation with Equal and Unequal intervals	2	2	30
III	Numerical Differentiation	1	1	15
IV	Numerical Integration	1	1	15
V	Numerical solution of ordinary differential equations	1	1	15
Total		7	6	95

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 3X10 = 30

.....
Total Marks = 50

.....

Pithapur Rajah's Government College (Autonomous), Kakinada

III Year B.Sc., Degree Examinations - V Semester

Mathematics Course VI A: NUMERICAL METHODS

Paper VI A (Model Paper w.e.f. 2023-24)

.....
Time: 2Hrs

ax. Marks: 50

SECTION-A

Answer Any Three Questions, Selecting At Least One Question from Each Part.

Part – A

3 X 10 = 30

1. Essay question from Unit - I.
2. Essay question from Unit – II
3. Essay question from Unit - II.

Part – B

1. Essay question from Unit - III .
2. Essay question from Unit - IV.
3. Essay question from Unit - V .

SECTION-B

Answer any four questions

4 X 5 M = 20 M

4. Short answer question from Unit - I.
5. Short answer question from Unit - I .
6. Short answer question from Unit – II.
7. Short answer question from Unit - II.
8. Short answer question from Unit - III.
9. Short answer question from unit – IV.
10. Short answer question from Unit – V.

PITHAPUR RAJAH'S GOVERNMENT COLLEGE (AUTONOMOUS), KAKINADA
DEPARTMENT OF MATHEMATICS

Question Bank for
PAPER-VI A: NUMERICAL METHODS

Short Answer Questions

Unit-I

1. Prove that i) $\Delta = E - 1$ ii) $\nabla = 1 - E^{-1}$
2. Prove that i) $(1 + \Delta)(1 - \nabla) = 1$ ii) $E\nabla = \Delta$ iii) $\Delta - \nabla = \Delta\nabla$
3. Prove that (i) $\mu^2 = 1 + \frac{\delta^2}{4}$, (ii) $\Delta = \frac{\delta^2}{2} + \delta \sqrt{1 + \frac{\delta^2}{4}}$
4. Prove that $e^x = \left(\frac{\Delta^2}{E}\right) e^x \cdot \frac{Ee^x}{\Delta^2 e^x}$, the interval of differencing being unity.
5. Given $y_0 = 3$, $y_1 = 12$, $y_3 = 81$, $y_4 = 100$. Find $\Delta^4 y_0$ without forming difference table
6. Prove that i) $u_3 = u_2 + \Delta u_1 + \Delta^2 u_0 + \Delta^3 u_0$ and ii) $u_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$.
7. Evaluate $\Delta^3(1 - x)(1 - 2x)(1 - 3x)$, the interval differencing being unity.
8. Find the missing term in the following data.

x	1	2	3	4	5	6	7
y	2	4	8	---	32	64	128

9. Find the missing term in the following data.

x	0	1	2	3	4
y	1	3	9	—	81

10. Compute $f(1.1)$ from the following table.

x	1	2	3	4	5
$f(x)$	7	12	29	64	123

Unit- II

11. Given that

x	50	51	52	53	54
-----	----	----	----	----	----

Tan x	1.1918	1.2349	1.2799	1.3270	1.3764
-------	--------	--------	--------	--------	--------

Using Gauss's backward formula , find the value of $\tan 51^{\circ}42'$.

12. Show that $f(x_0, x_1, x_2, \dots, x_n) = \frac{\Delta^n f(x_0)}{n!h^n}$.

13. Construct a divided difference table for the following .

x	1	2	4	7	12
f(x)	22	30	82	106	216

14. Find the third divided difference with arguments 2 , 4 , 9 , 10 of the function $f(x) = x^3 - 2x$.

15. Derive Lagrange's interpolation formula.

16. By Lagrange's interpolation formula , find the value of y at $x = 5$, given that

x	1	3	4	8	10
f(x)	8	15	19	32	40

17. By Lagrange's interpolation formula , find the form of the function given by

x	0	1	2	3	4
f(x)	3	6	11	18	27

UNIT - III

18. Find the first order derivative of $\sqrt{x}$ at $x = 15$ from the following .

x	15	17	19	21	23	25
f(x)	3.8773	4.123	4.359	4.583	4.796	5.000

19. Find $f'(1)$ for $f(x) = \frac{1}{1+x^2}$ using the following table .

x	1.0	1.1	1.2	1.3	1.4
f(x)	0.5000	0.4524	0.4098	0.3717	0.3378

20. Find $f'(1.5)$ from the following table .

x	0.0	0.5	1.0	1.5	2.0
---	-----	-----	-----	-----	-----

f(x)	0.3989	0.3521	0.2420	0.1245	0.0540
-------	--------	--------	--------	--------	--------

21. Find $f'(5)$ from the following table .

x	1	2	4	8	10
f(x)	0	1	5	21	27

UNIT - IV

22. Evaluate $\int_0^1 (4x - 3x^2) dx$ taking 10 intervals by Trapezoidal rule .

23. Evaluate $\int_0^{\pi} t \sin t dt$ by using Trapezoidal rule .

24. Calculate the approximate value of $\int_{-3}^3 x^3 dx$ by using Trapezoidal Rule .

25. Evaluate $I = \int_0^1 \frac{dx}{1+x}$ correct to three decimal places by Trapezoidal rule with $h = 0.25$

26. Evaluate the integral $\int_1^2 \sqrt{1 - \frac{1}{x}} dx$ by Simpson's 1/3 rule with five ordinates .

27. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Weddle's Rule .

UNIT - V

28. Solve the differential equations $\frac{dy}{dx} = x + y$, with $y(0) = 1$, $x \in [0,1]$ by Taylor series expansion to obtain y for $x = 0.1$.

29. Using the Taylor's series for $y(x)$, find $y(0.1)$ correct to four decimal places if $y(x)$ satisfies $y' = x - y^2$, $y_0 = 1$ where $x_0 = 0$.

30. Solve $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$ by Picard's method .

31. Given $\frac{dy}{dx} = y + x^3$, $y(0) = 1$, compute $y(0.02)$ by Euler's method taking $h = 0.01$.

32. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y = 1$ when $x = 0$, find y for $x = 0.1$ in 4 steps by Euler's method

Essay Questions

Unit-I

1. Show that $\Delta^n \cos(ax + b) = (2\sin \frac{ah}{2})^n \cos[a + bx + n(\frac{ah+\pi}{2})]$
2. State and prove fundamental theorem of difference calculus .
3. State and prove Newton's – Gregory formula for forward interpolation with equal intervals .
4. The area of a circle of diameter d is given for the following values , find the approximate value for the area of a circle of diameter 82 .

d(Diameter)	80	85	90	95	100
A(Area)	5026	5674	6362	7088	7854

5. From the following table , find the number of students who obtain less than 56 marks

Marks	30-40	40-50	50-60	60-70	70-80
No.of students	31	42	51	35	31

6. State and prove Newton's – Gregory backward interpolation formula with equal intervals .
7. The population of a country in the decennial census were as under . Estimate the population for the year 1925 .

Year(x)	1891	1901	1911	1921	1931
Population(y) (in thousands)	46	66	81	93	101

8. Given

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

Find f(7.5)

UNIT - II

9. Using Gauss forward formula find u_{32} from the given data $u_{20} = 14.035$, $u_{25} = 13.674$, $u_{30} = 13.257$, $u_{35} = 12.734$, $u_{40} = 12.089$, $u_{45} = 11.309$.
10. Apply Gauss forward formula to find the value of u_9 if $u_0 = 14$, $u_4 = 24$, $u_8 = 32$, $u_{16} = 40$.
11. Interpolate by means of Gauss backward interpolation formula the sales for the concern for the year 1936, given that

<i>year</i>	1901	1911	1921	1931	1941	1951
<i>sales(in thousands)</i>	12	15	20	27	39	52

12. Given that $\sqrt{12500} = 111.803399$, $\sqrt{12510} = 111.848111$, $\sqrt{12520} = 111.892806$, $\sqrt{12530} = 111.937483$, show $\sqrt{12516} = 111.8749301$ by using Gauss backward interpolation formula.

13. Apply Stirling's formula to find y_{28} given that $y_{20}=49225$, $y_{25} = 48316$, $y_{30} = 47236$, $y_{35} = 45926$, $y_{40} = 44300$.

14. Given $y_{20} = 24$, $y_{24} = 32$, $y_{28} = 35$, $y_{32} = 40$, find y_{25} by Bessel's formula.

15. State and prove Newton's divided difference formula.

16. By means of Newton's divided difference formula, find the values of $f(8), f(15)$ from the following table.

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

17. Using Lagrange's interpolation formula find y at $x = 301$.

X	300	304	305	307
Y	2.4771	2.4829	2.4843	2.4871

18. Using Lagrange's interpolation formula, prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8} \left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3}) \right]$$

UNIT - III

19. Using the following table, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1$.

x	1	2	3	4	5	6
y	1	8	27	64	125	216

20. Using the following table, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.2$.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

21. Using the following table, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2.2$.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
$f(x)$	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

22. Find $f'(0.6)$ and $f''(0.6)$ from the following table .

x	0.4	0.5	0.6	0.7	0.8
f(x)	1.5836	1.7974	2.0442	2.3275	2.6510

23. Find the maximum and minimum values of the function $y = f(x)$ from the following table .

x	0	1	2	3	4	5
f(x)	0	0.25	0	2.25	16	56.25

UNIT - IV

24. State and prove Simpson's 1/3 rule .

25. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's 1/3 rule .

26. State and prove Simpson's 3/8 rule .

27. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's 1/3 and 3/8 rule . Hence obtain the approximate value of π in each case .

28. Integrate numerically $\int_4^{5.2} \log x \, dx$ by Weddle's rule .

29. Evaluate the integral $\int_0^{\frac{\pi}{2}} \sqrt{\sin x} \, dx$ by Weddle's rule .

UNIT - V

30. Determine the value of y when $x = 0.1$ given that $y(0) = 1$ and $y' = x^2 + y$ by Euler's modified method .

31. Find $y(0.2)$ by using Euler's modified method for $\frac{dy}{dx} = \log_{10}(x + y)$ with initial condition $y = 1$ for $x = 0$.

32. Solve $\frac{dy}{dx} = xy$ using Runge – Kutta method for $x = 0.2$ given that $y(0) = 1$ taking $h = 0.2$.

33. Given $\frac{dy}{dx} = y - x$ with $y(0) = 2$ find $y(0.1)$ and $y(0.2)$ correct to four decimal place by R – K method .

34. Apply Runge-Kutta method find the solution of the differential equation $y' = 3x + y/2$ with $y_0 = 1$ at $x = 0.1$.

